A Dynamic Evacuation Model

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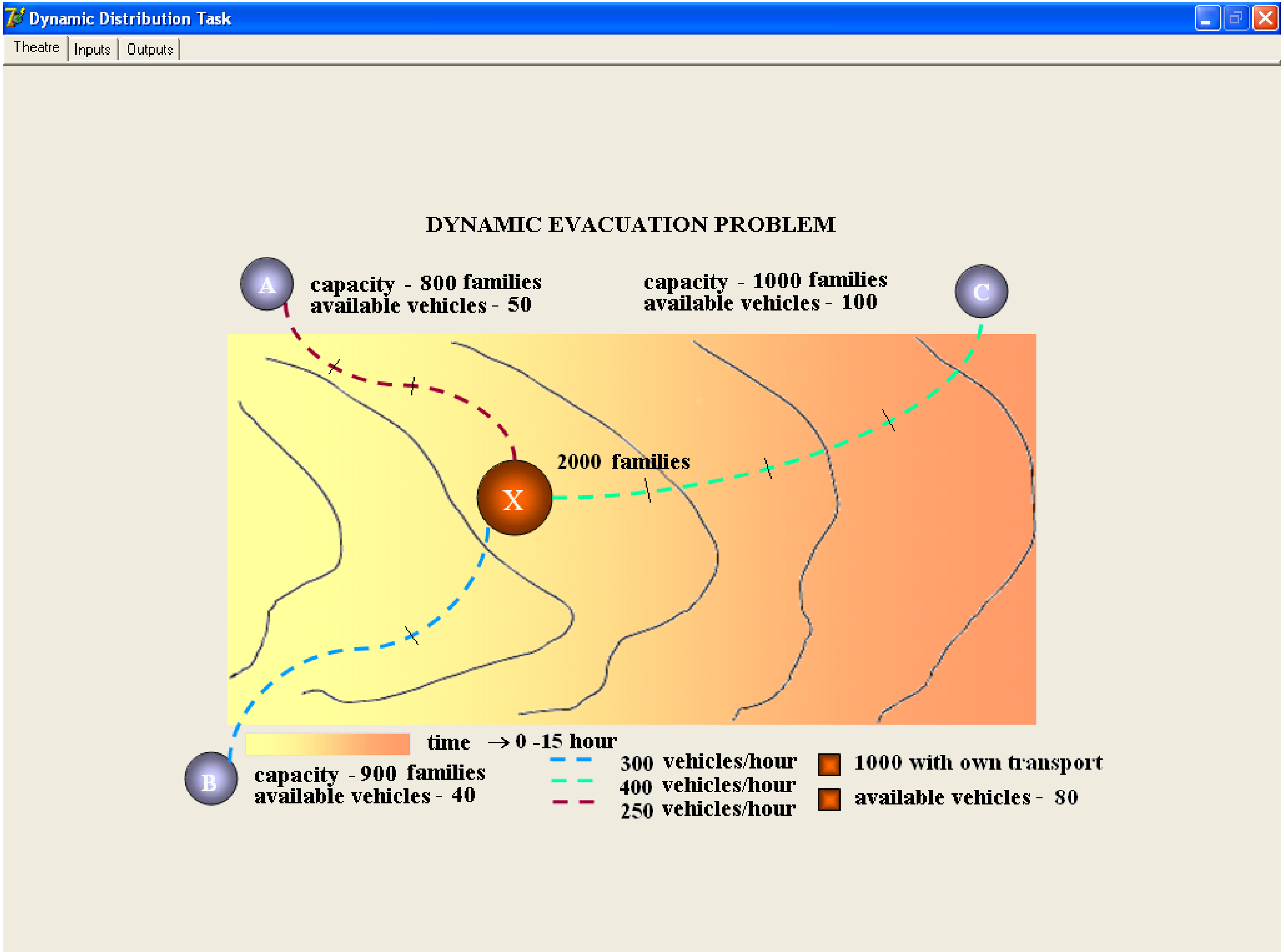
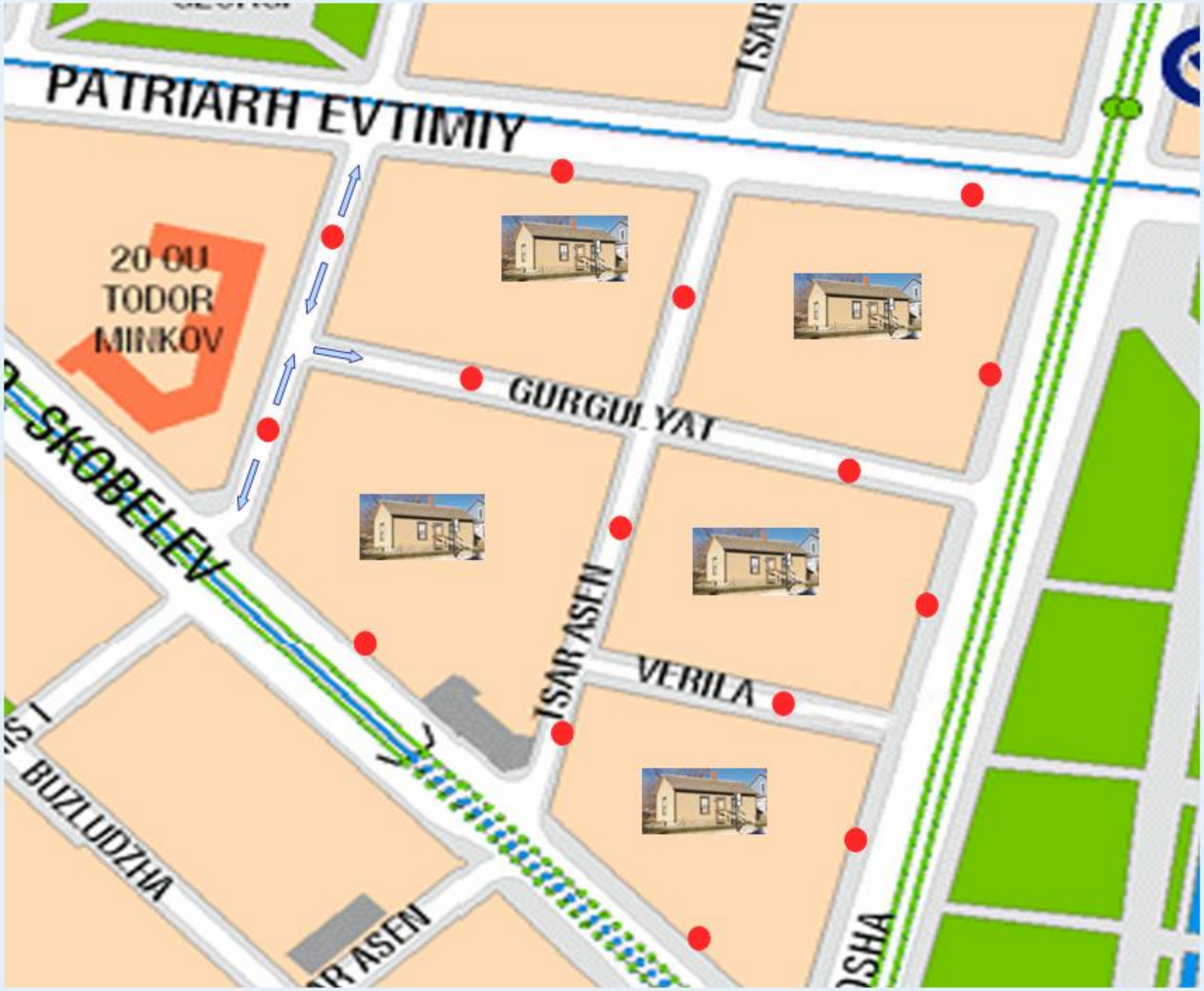
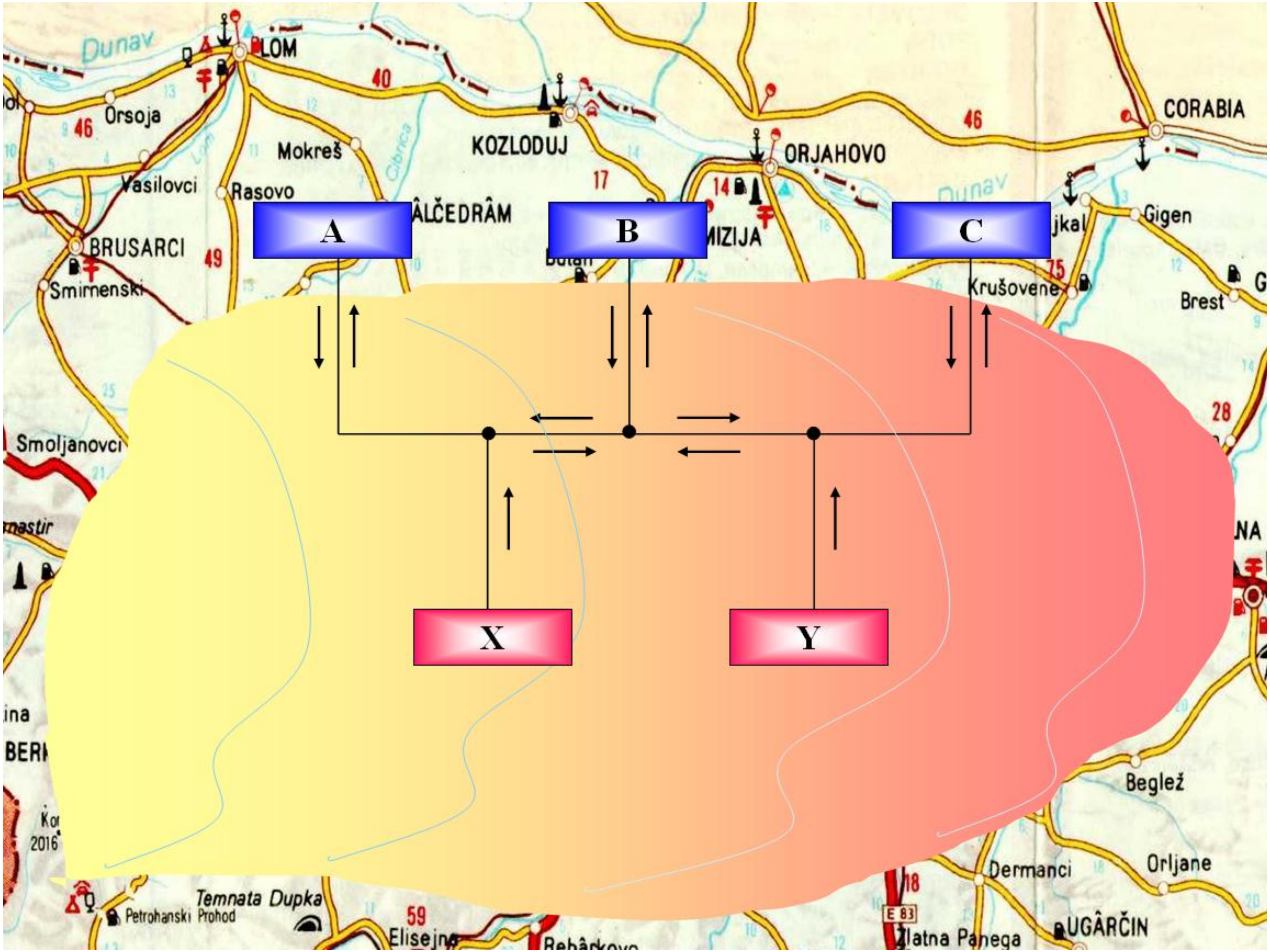

Dynamic Distribution Task
Theatre
Inputs
Outputs
DYNAMIC EVACUATION PROBLEM
A
capacity - 800 families
available vehicles - 50
capacity - 1000 families
available vehicles - 100
C
2000 families
X
time → 0 -15 hour
B
capacity - 900 families
available vehicles - 40
300 vehicles/hour
400 vehicles/hour
250 vehicles/hour
1000 with own transport
available vehicles - 80


PATRIARH EVTIMIY
20 OU
TODOR
MINKOV
GURGULYAT
SKOBELEV
TSAR ASEN
VERILA
BUZLUDZHA


A
B
C
X
Y
Dunav
LOM
Orsoja
Mokreš
KOZLODUJ
ORJAHOVO
CORABIA
Vasilovci
Rasovo
BRUSARCI
Smirnenski
MIZIJA
Gigen
Krušovene
Brest
Smoljanovci
Beglež
Dermanci
Orljane
Temnata Dupka
Petrohanski Prohod
Zlatna Panega
UGÂRČIN
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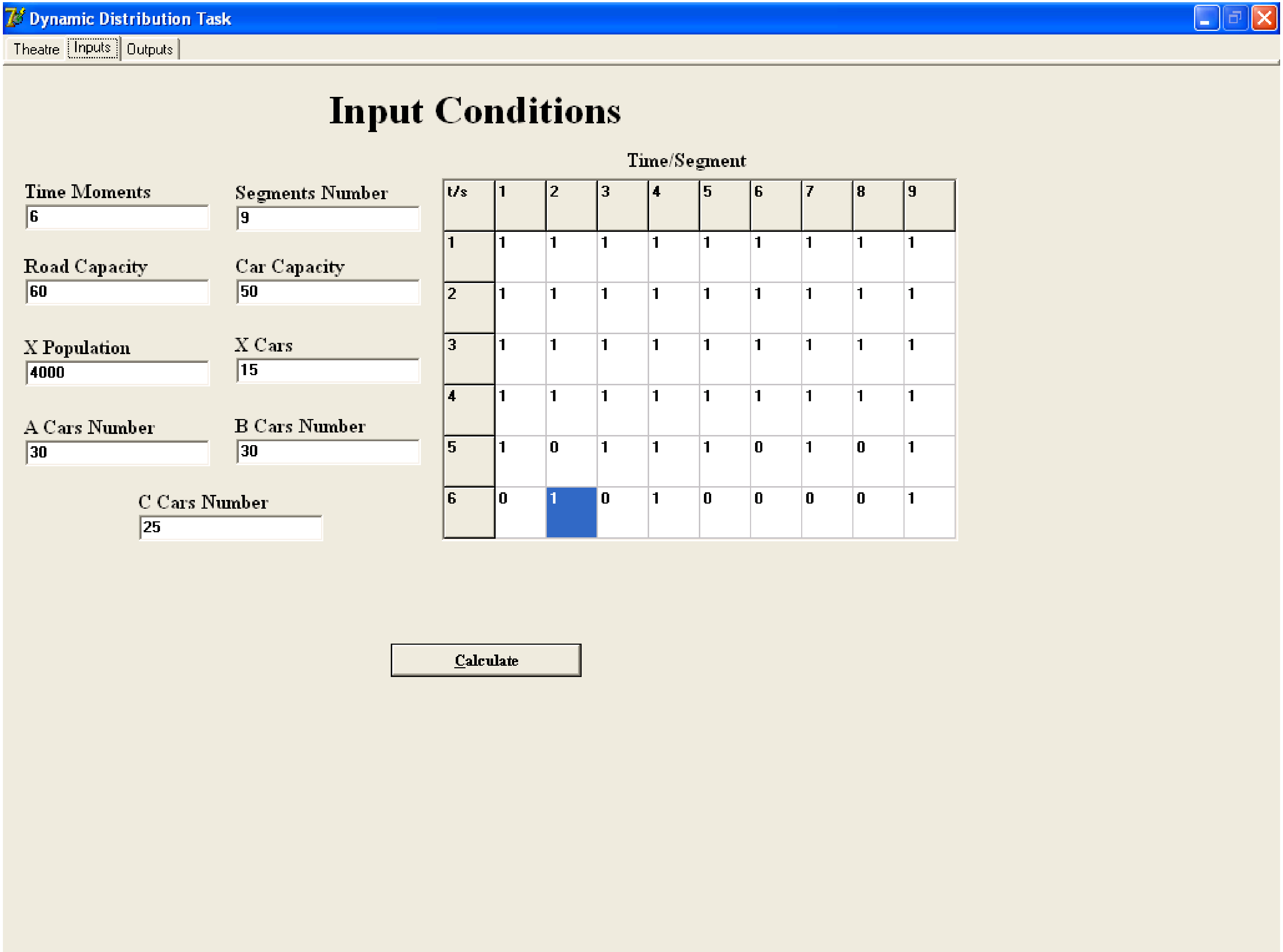
Dynamic Distribution Task
Theatre
Inputs
Outputs
Input Conditions
Time Moments
6
Segments Number
9
Road Capacity
60
Car Capacity
50
X Population
4000
X Cars
15
A Cars Number
30
B Cars Number
30
C Cars Number
25
Time/Segment
t/s 1 2 3 4 5 6 7 8 9
1 1 1 1 1 1 1 1 1 1
2 1 1 1 1 1 1 1 1 1
3 1 1 1 1 1 1 1 1 1
4 1 1 1 1 1 1 1 1 1
5 1 0 1 1 1 0 1 0 1
6 0 1 0 1 0 0 0 0 1
Calculate

Dynamic Distribution Task

Theatre | Inputs | Outputs

Output Results

Send at time t to town Y

t/Y	A	B	C
1	0	750	0
2	0	1500	0
3	0	1250	0
4	0	0	0
5	0	0	500
6	0	0	0

Call at time t from town Y

t/Y	A	B	C
1	25	30	0
2	0	0	10
3	0	0	0
4	0	0	0
5	0	0	0
6	0	0	0

Pop. in X at time t

t/X	X
1	3250
2	1750
3	500
4	500
5	0
6	0

Travelling to Y = [A, B, C] at moment t=6

A	0	0	0

B	0	0

C	0	500	0	0

The Evacuation Problem in Continuous Time

- Notations

T - the time at which the disaster will hit town X

$p(t)$ - number of families in town X at time t

$u_1(t)$ - number of families sent to town A at time t

$u_2(t)$ - number of families sent to town B at time t

$u_3(t)$ - number of families sent to town C at time t

$v_1(t)$ - number of transport vehicles sent to town A at time t

$v_2(t)$ - number of transport vehicles sent to town B at time t

$v_3(t)$ - number of transport vehicles sent to town C at time t

$y_1(t)$ - number of transport vehicles called from town A at time t

$y_2(t)$ - number of transport vehicles called from town B at time t

$y_3(t)$ - number of transport vehicles called from town C at time t

$\diamondsuit$ To be calculated from the forecast:

$t_1(x)$ - the time at which the point on the road from X to A at distance x from X, will be reached by the disaster

$t_2(x)$ - the time at which the point on the road from X to B at distance x from X, will be reached by the disaster

$t_3(x)$ - the time at which the point on the road from X to C at distance x from X, will be reached by the disaster

◊ Other data assumed known:

Every transport vehicle can take r families

q families have their own transport

At the beginning there are M transport vehicles in town X as well as N_1, N_2 and N_3 transport vehicles in towns A, B and C respectively

The capacities of the roads to A, B and C are c_1, c_2 and c_3 respectively (number of vehicles per unit of time)

The distances between town X and towns A, B and C are a_1, a_2 and a_3 respectively

The velocities on the roads from X to A, B and C are v_1, v_2 and v_3 respectively

The towns A, B and C can accommodate m_1, m_2 и m_3 families respectively

- More notations

$*$ t_i is the solution to the problem

$$\min_{0 \le x \le a_i} \left\{ t_i(x) - \frac{x}{v_i} \right\}$$

for $i = 1, 2, 3$

$*$ if $t_i \le 0$, we set $c_i(t) = 0$ for each $t \in [0, T]$

$*$ if $t_i > 0$, we set

$$c_i(t) = \begin{cases} c_i & \text{за} \quad t \le t_i \\ 0 & \text{за} \quad t > t_i \end{cases}$$

* $\bar{t}_i$ is the solution to the problem

$$\min_{0 \le x \le a_i} \left\{ t_i(x) - \frac{a_i - x}{v_i} \right\}$$

for $i = 1, 2, 3$

* if $\bar{t}_i \le 0$, we set $\bar{c}_i(t) = 0$ for each $t \in [0, T]$

* if $\bar{t}_i > 0$, we set

$$\bar{c}_i(t) = \begin{cases} c_i & \text{за} \quad t \le \bar{t}_i \\ 0 & \text{за} \quad t > \bar{t}_i \end{cases}$$

- state equation

$$\dot{p}(t) = -u_1(t) - u_2(t) - u_3(t) - rv_1(t) - rv_2(t) - rv_3(t)$$

- initial condition

$$P(0) = p_0$$

- Constraints

∗ from the roads' capacities

$$0 \leq u_i(t) + 3.v_i(t) \leq c_i(t)$$
$$0 \leq y_i(t) \leq \bar{c}_i(t)$$

∗ from the availability (or lack) of own transport

$$\int_0^T (u_1(t) + u_2(t) + u_3(t)) \, dt \leq q$$
$$\int_0^T (v_1(t) + v_2(t) + v_3(t)) \, dt \leq (p_0 - q)/r$$

* from the accommodation capacities of A, B and C

$$\int_0^T (u_1(t) + rv_1(t))\,dt \leq m_1$$
$$\int_0^T (u_2(t) + rv_2(t))\,dt \leq m_2$$
$$\int_0^T (u_3(t) + rv_3(t))\,dt \leq m_3$$

* from the availability of transport vehicles in A, B and C respectively

$$\int_0^T y_1(t)\,dt \leq N_1$$
$$\int_0^T y_2(t)\,dt \leq N_2$$
$$\int_0^T y_3(t)\,dt \leq N_3$$

* from the availability of transport vehicles in X

- for $t \in [0, 2)$

$$\int_0^t (v_1(\tau) + v_2(\tau) + v_3(\tau))\, d\tau \leq M$$

- for $t \in [2, 3)$

$$\int_0^t (v_1(\tau) + v_2(\tau) + v_3(\tau))\, d\tau \leq M + \int_0^{t-2} y_2(\tau)\, d\tau$$

- for $t \in [3, 4)$

$$\int_0^t (v_1(\tau) + v_2(\tau) + v_3(\tau))\, d\tau \leq M + \int_0^{t-2} y_2(\tau)\, d\tau + \int_0^{t-3} y_1(\tau)\, d\tau$$

- ○ for $t \in [4, T]$

$$\int_0^t (v_1(\tau) + v_2(\tau) + v_3(\tau))\; d\tau \;\leq\; M + \int_0^{t-2} y_2(\tau)\; d\tau + \int_0^{t-3} y_1(\tau)\; d\tau + \int_0^{t-4} y_3(\tau)\; d\tau$$

- objective function

$$p(T) \longrightarrow \min$$

- objective function

$$p(T) \longrightarrow \min$$