Mathematical Models for Force Planning

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Given is a comprehensive list of (situational) defence scenarios, each of them posing specific threats/challenges.

- the political leadership determines the missions of the armed forces (e.g. defend the sovereignty and the territorial integrity of the country, contributing to the international piece, etc.)
- a list of military tasks is derived from the missions (e.g. defend the national territory, defend the sovereignty of the national airspace, etc.)
- the comprehensive list of defence scenarios is developed to present all conceivable (according to the planners) threats/challenges the armed forces would have to deal with in the planning period and within the politically determined level of ambitions
- the scenarios are developed to describe all conceivable situations in which the military tasks will have to be fulfilled

What is a situational scenariao: an example

Dirty bomb with suicide aircraft (terrorist attack)

On Sunday, October 7, the soccer derby between the eternal arch-rivals "Levski" and CSKA is being played in the National Stadium in Sofia, starting at 17:00.

There are about 25 000 people attending.

At 16:55 a small private aircraft takes off from Bojourishte airfield some 30 km north-west of Sofia.

At 17:10 the aircraft flying at low altitude reaches the stadium from its western side, crashes into the seats of sector B and explodes. (Big explosion with flames and cloud, large amount of casualties: 90 people killed, about 230 injured.)

At 17:10 crowd movements in the stadium begin:
- A lot of people die (more than 100) or are injured due to the panic movements;
- TV cameras are covering the scene;
- Immediate reaction of security forces and Crisis Management Centre;

At 17:40 the stadium is evacuated, first responders arrive on the explosion site;

At 17:52 radiological material is detected by the first responders team, alert to the Crisis Management Centre;

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A similar example (**A VIP event**)

On Friday, October 12, a one-day conference of the EU foreign ministers is taking place in "Hilton" hotel in Sofia. The conference starts at 10:00.

The foreign ministers of all 27 member countries are attending.

At 10.05 a small private aircraft takes off from an airfield near the town of Elin Pelin some 30 km east of Sofia.

At 10:20 the aircraft, packed with explosives, flying at low altitude reaches the hotel from its eastern side, crashes into it and explodes.

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A list of unit types (called "generic units") is also given.

- in the airforce these are tipically
  - a battle aircraft (a bomber, a fighter, etc.)
  - a transport aircraft
  - a communication system (command and control)
  - personnel
  - . . . . . .
- in the navy these are tipically
  - a battleship (many types of them)
  - a transport vessel
  - a communication system (command and control)
  - personnel
  - . . . . . .

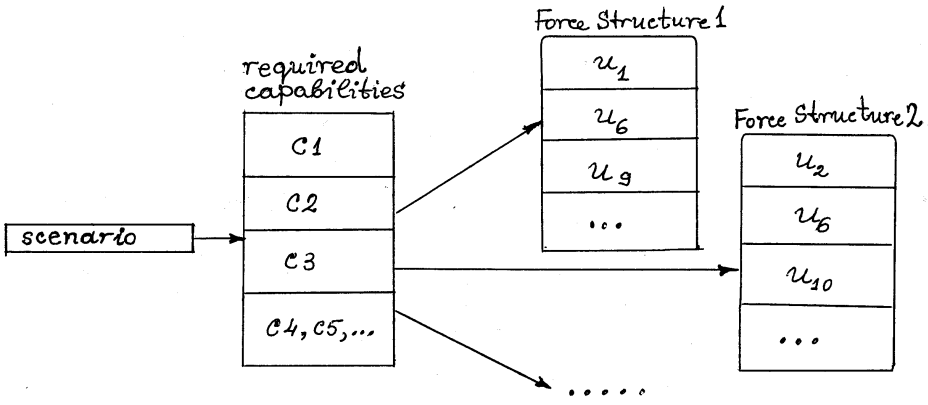
- Various force structures can be formed by units from different types.
- A list of capabilities having measurable (quantifiable) levels is determined by the whole set of scenarios.
- Each force structure determines a package of capability levels (a vector of capabilities).

We aim at presenting an approach for optimizing the force structure in order to better neutralize the threats subject to constraints.

The scenario - force structure multivalued map:

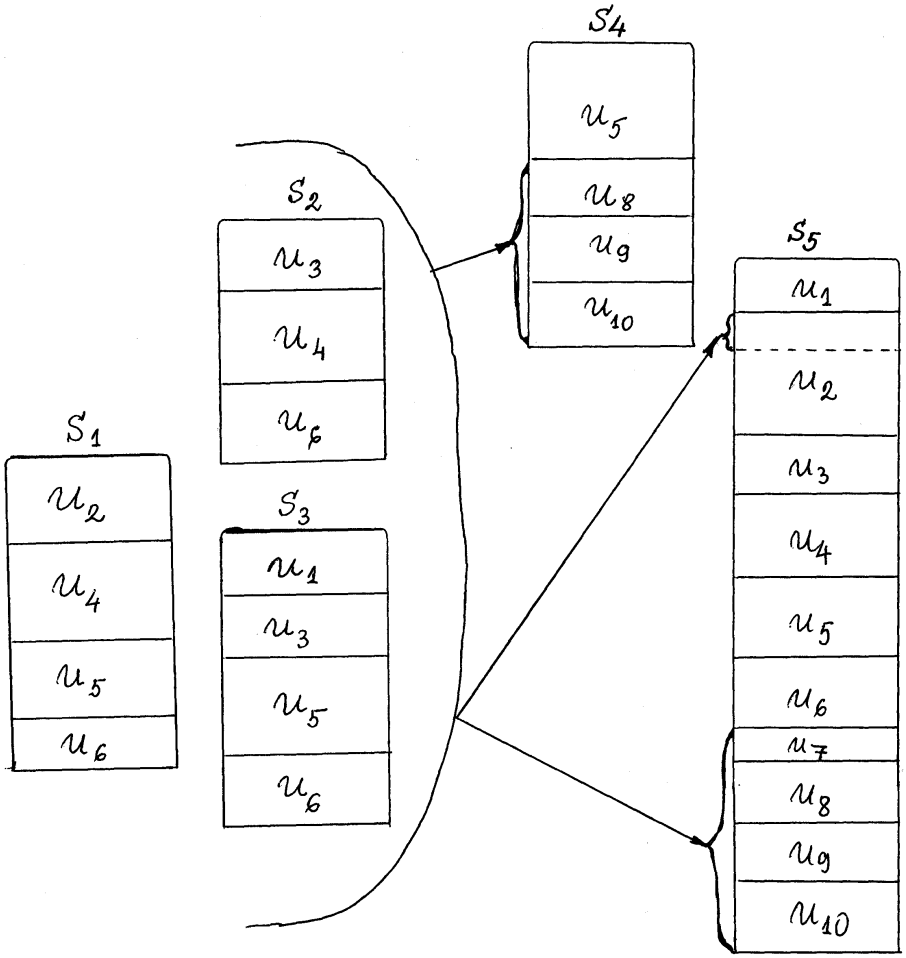
- Capability 1 - detect the threat early enough
  - ground based radar (capable of detecting low flying targets)
  - AWACS aircraft (Airborne Warning and Control System)
  - sattelite surveillance
  - . . . . . .
- Capability 2 - eliminate the threat
  - ground-to-air guided missiles system
  - air patrolling during the event
    - jet-fighters
    - combat helicopters
  - . . . . . .

- Capability 3 - exercise efficient command and control (in case of a hijacked passenger plane, directed to the nuclear powerplant, a decision will have to be taken at the top political level immediately)
  - communication system type A
  - communication system type B
  - . . . . . .
- Capabilities 4,5,... - Consequences Management
  - Capabilities in health care
  - Capability in putting out fires
  - Capability in detecting radiological contamination and in eliminating it
  - . . . . . .


scenario
required capabilities
C1
C2
C3
C4, C5, ...
Force Structure 1
u_1
u_6
u_9
...
Force Structure 2
u_2
u_6
u_{10}
...
.....

- A force structure is related to each scenario in such a way that it can neutralize the negative effects of the scenario.
- The force sructure has the property that if the number of any type of units in it is decreased, the negative effects may not be neutralized.
- The price of one unit of any type of force units is given (determined for a specified time period).

Define the **distance** from a given set of scenarios to a particular scenario to be the total price of those units of the force structure related to this particular scenario, which are not present in all the force structures necessary to neutralize the scenarios from the given set.


S_1
u_2
u_4
u_5
u_6
S_2
u_3
u_4
u_6
S_3
u_1
u_3
u_5
u_6
S_4
u_5
u_8
u_9
u_{10}
S_5
u_1
u_2
u_3
u_4
u_5
u_6
u_7
u_8
u_9
u_{10}

With a given budget and given prices of one unit of any of the unit-types we want to choose several from all the situational scenarios in such a way that the cost of the force structure for all the chosen scenarios does not exceed the budget and the sum of all distances from the set of the chosen scenarios to any of the non-chosen scenarios be miniminal.

or

With a given budget and given prices of one unit of any of the unit-types we want to choose several from all the situational scenarios in such a way that the cost of the force structure for all the chosen scenarios does not exceed the budget and the maximal distance from the set of the chosen scenarios to any of the non-chosen scenarios be miniminal.

These two are integer linear programming problems.

The notations:

n – the number of all situational scenarios;
m – the number of all types of generic units;
l_{ik} – the necessary number of the k–type units in the i–th scenario;
p_k – the price of one unit of the k–type units;
x_i – binary variable, equals 0 if the i–th scenario is not chosen and equals 1 if the i–th scenario is chosen;

z_{ik} – a variable, shows by how much the number of necessary units of the k–type in the i–th scenario exceeds the total number of the k–type units in all chosen scenarios;
b – the available budget;
v – artificial variable;

$$\sum_{i=1}^{n}\sum_{k=1}^{m} p_k z_{ik} \longrightarrow \text{ minimize}$$

subject to the constraints

$$z_{ik} \geq l_{ik} - \sum_{j=1}^{n} x_j l_{jk} \text{ for } i = 1, 2, \ldots, n; k = 1, 2, \ldots, m;$$

$$\sum_{i=1}^{n} x_i \sum_{k=1}^{m} p_k l_{ik} \leq b;$$

$$x_i \in \{0, 1\} \text{ for } i = 1, 2, \ldots, n;$$

$$z_{ik} \geq 0 \text{ for } i = 1, 2, \ldots, n; k = 1, 2, \ldots, m.$$

$$v \longrightarrow \text{ minimize}$$

subject to the constraints

$$z_{ik} \geq l_{ik} - \sum_{j=1}^{n} x_j l_{jk} \text{ for } i = 1, 2, \ldots, n; k = 1, 2, \ldots, m;$$

$$\sum_{i=1}^{n} x_i \sum_{k=1}^{m} p_k l_{ik} \leq b;$$

$$\sum_{k=1}^{m} p_k z_{ik} \leq v \text{ for } i = 1, 2, \ldots, n;$$

$$x_i \in \{0, 1\} \text{ for } i = 1, 2, \ldots, n;$$

$$z_{ik} \geq 0 \text{ for } i = 1, 2, \ldots, n; k = 1, 2, \ldots, m.$$

Let N_i be the negative consequences (measured, e.g., in financial terms) from scenario i if we are not prepared to confront it. Then we have the following problem:

$$\sum_{i=1}^{n}(1-x_i)N_i \longrightarrow \text{ minimize } \left(\sum_{i=1}^{n} x_i N_i \longrightarrow \text{ maximize}\right)$$

subject to the constraints

$$\sum_{i=1}^{n} x_i \sum_{k=1}^{m} p_k l_{ik} \leq b;$$

$$x_i \in \{0,1\} \text{ for } i = 1, 2, \ldots, n;$$

Let the vector $(u_1, u_2, \ldots, u_m)$ of units of all unit types be given. The next problem consists in distributing these units to those among all scenarios so that the total negative effect be minimized:

$$\sum_{i=1}^{n}(1 - x_i)N_i \longrightarrow \text{ minimize } \left(\sum_{i=1}^{n} x_i N_i \longrightarrow \text{ maximize}\right)$$

subject to the constraints

$$\sum_{i=1}^{n} x_i l_{ik} \leq u_k \text{ for } k = 1, 2, \ldots, m;$$

$$x_i \in \{0, 1\} \text{ for } i = 1, 2, \ldots, n;$$

Denote the optimal value of the obgective function by $N(u_1, u_2, \ldots, u_m)$.

Dynamic Framework

We have a planning horizon T units of time long, actions are taken at $t = 0, 1, 2, \ldots, T$.

We have an initial force structure $(u_{1,0}, u_{2,0}, \ldots, u_{m,0})$ at $t = 0$.

We want to reach force structure $(\bar{u}_1, \bar{u}_2, \ldots, \bar{u}_m)$ at $t = T$.

Let

$p_{k,t}$ be the cost of maintaining one unit of type k at time t

for $k = 1, 2, \ldots, m$, $t = 0, 1, 2, \ldots, T$;

$\bar{p}_{k,t}$ be the cost of acquiring one additional unit of type k at time t

for $k = 1, 2, \ldots, m$, $t = 0, 1, 2, \ldots, T$;

$\tilde{p}_{k,t}$ be the cost of decreasing by one the number of units of type k at time t for $k = 1, 2, \ldots, m$, $t = 0, 1, 2, \ldots, T$;

Denote by $P(t)$ the cost incurred at time t for $t = 0, 1, 2, \ldots, T$:

$$\sum_{k=1}^{m} p_{k,t} u_{k,t} + \sum_{k=1}^{m} \max\{0, u_{k,t+1} - u_{k,t}\} \bar{p}_{k,t} + \sum_{k=1}^{m} \max\{0, u_{k,t} - u_{k,t+1}\} (\tilde{p}_{k,t} - p_{k,t})$$

Let b_t be the available budget at time t for $t = 0, 1, 2, \ldots, T$.

$$\max_{1\le t\le T} N(u_{1,t}, u_{2,t}, \ldots, u_{m,t}) \longrightarrow \text{ minimize}$$

subject to the constraints

$$P(t) \le b_t \quad \text{for} \quad t = 0, 1, 2, \ldots, T;$$

$$0 \le u_{k,t} \quad \text{for} \quad k = 1, 2, \ldots, m, \quad t = 0, 1, 2, \ldots, T;$$

$$u_{k,t} \quad \text{integer for} \quad k = 1, 2, \ldots, m, \quad t = 0, 1, 2, \ldots, T;$$

$$u_{k,0} \quad \text{given for} \quad k = 1, 2, \ldots, m;$$

$$u_{k,T} = \bar{u}_k \quad \text{for} \quad k = 1, 2, \ldots, m.$$