NATO Science for Peace and Security Series
D: Information and Communication Security - Vol. 12

Scientific Support for the Decision Making in the Security Sector

Edited by
Ognyan Kounchev
Rene Willems
Velizar Shalamanov
Tsvetomir Tsachev

IOS
Press

NATO
OTAN

This publication is supported by: The NATO Science for Peace and Security Programme

Dynamic Evacuation Models

TSVETOMIR TSACHEV

Institute of Mathematics and Informatics - Bulgarian Academy of Sciences

ABSTRACT. We present four deterministic models of evacuation activities emphasizing the dynamical element in their organization. Both discrete time and continuous time models are presented. The discrete time model is in the form of a linear programming problem. The three continuous time models are in the form of optimal control problems. The need to find quantitative description of the various behavioural patterns during large scale evacuations is discussed.

KEYWORDS: Dynamic evacuation, optimal control, linear programming

1. Introduction.

The problem of controlling dynamical flows on networks arises in numerous applications of mathematical techniques to decision making support. In this paper we propose such an application to the organization of evacuation activities. The importance of such studies reemerged after hurricane Katrina hit the Gulf Coast of the US in August 2005. From then on we have witnessed a number of extreme weather phenomena, some of them also in Europe, like the floods in the summer of 2005 and hurricane Kyrill in January 2007. There may be other causes for large numbers of people to be moved away from their places of residence, like terrorist activities or industrial failures.

In the present paper we approach the problem of organizing evacuation activities from a deterministic point of view. We consider the problem of evacuation of a residential area caused by a hazardous phenomenon (flood, poisonous or radioactive cloud, storm, etc.) approaching this area. Assuming complete knowledge of the dynamics of the hazardous phenomenon we propose four models of the evacuation. In two of them we further assume that the local authorities can fully control the traffic flows of evacuees by telling them who should leave when and who should go where.

The linear problems we formulate in Sections 2 and 3 have the advantage of being numerically tractable. Of course, many mathematical models are in the form of nonlinear problems. In our case, if we have reliable knowledge about nonlinearities in the evacuation activities, we can build nonlinear models, too. This can be done both in the discrete time and in the continuous time set up. We propose two nonlinear continuous time models in Section 4. All the three continuous time models in this paper are formulated as optimal control problems. There are many textbooks on the topic, [3] can serve as a good introduction.

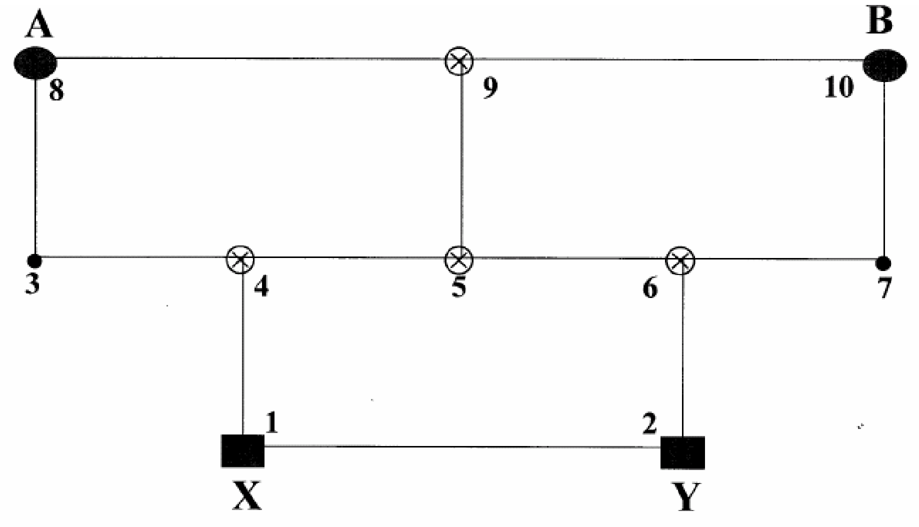
By now there is vast literature on organizing evacuation activities and the problems are tackled from various points of view, the reader is referred to [1] and [4] and the references therein for a quick overview of some recent contributions. The paper [4] reviews the evolution of evacuation plans for the city of New Orleans over the last decade, pointing out to some successes and some failures of the evacuation caused by hurricane Katrina in August 2005. In [1] the authors explore options that allow evacuees flexibility in selecting their exit routes and destinations. They present a case study, which simulates a country-wide evacuation operation using *DYNASMART-P* [2] for Knox County, Tennessee, comparing eight scenarios that represent various destination and route-assignment strategies. In the present paper we approach the considered problems from a theoretical viewpoint, the main emphasis being the building of the mathematical models.

2. An Evacuation Problem in Discrete Time

In this section we formulate a simple evacuation problem in discrete time.

Let us assume that we have precise information about the dynamics of a hazardous phenomenon (flood, harmful cloud, etc.) in a given time interval $[0,T]$. This means we assume that we can exactly predict its location (its outer bounds, its front) in every of the discrete time moments $t = 0,1,2,...,T$. We further assume that the two towns X and Y are in the disaster zone and their population must be evacuated. There are two nearby locations, towns A and B, which are not expected to be hit by the disaster and where the population of X and Y will be directed for temporary shelter. Let us assume that the available for the evacuation road infrastructure is given by the graph on the Figure.

The evacuation activities are planned up till the given moment of time T - the end of the time horizon. Actions are taken in every of the discrete time moments $t = 0,1,2,...,T$. Let us denote by $\overline{t_X}$ and by $\overline{t_Y}$ the time when towns X and Y respectively are hit by the disaster.



Let $X(t)$ denote the number of people at time t in town X and $Y(t)$ denote the number of people at time t in town Y. Let $A(t)$ denote the number of evacuees at time t in town A and $B(t)$ denote the number of evacuees at time t in town B. The initial conditions are given: $X(0), Y(0)$, $A(0)=0$ and $B(0)=0$.

By $u_{ij}(t)$ we denote the number of people sent from node i to node j at time t - these are the decision (control) variables. They must satisfy the constraints

$$0 \le u_{ij}(t) \le c_{ij}(t).$$

If the arc between nodes i and j is not cut off by the hazardous phenomenon at time t, we set $c_{ij}(t)=c_{ij}$, where c_{ij} denotes the capacity of the arc (the road) between nodes i and j. If this arc is cut by the disaster (no flow of evacuees can get through it) at time t, we set $c_{ij}(t)=0$. Obviously $c_{ij}(t)=c_{ji}(t)$ must hold.

There are three types of nodes in the graph.

The first type of nodes is represented by the nodes X, Y, A and B. They are characterized by the fact that they can serve as "reservoirs" - at any moment of time t the inflow need not equal the outflow. The respective equations for the four nodes are:

$$X(t+1) = X(t) - u_{12}(t) - u_{14}(t) + u_{21}(t) + u_{41}(t),$$
$$Y(t+1) = Y(t) - u_{21}(t) - u_{26}(t) + u_{12}(t) + u_{62}(t),$$
$$A(t+1) = A(t) - u_{83}(t) - u_{89}(t) + u_{38}(t) + u_{98}(t),$$
$$B(t+1) = B(t) - u_{10,7}(t) - u_{10,9}(t) + u_{7,10}(t) + u_{9,10}(t).$$

These equations have to be satisfied for each $t \in \{0,1,2,\ldots,T-1\}$. Of course, one has to take into account the nonnegativity constraints $A(t) \geq 0$, $B(t) \geq 0, X(t) \geq 0$ and $Y(t) \geq 0$ for $t \in \{0,1,2,\ldots, T\}$. Also, let us assume that due to the available accommodation capacity in A and B, we have the constraints $A(t) \leq A_{max}$ and $B(t) \leq B_{max}$ for $t \in \{1,2,3,\ldots, T\}$.

It is immediately seen that the variables $A(t)$, $B(t), X(t)$ and $Y(t)$ can be expressed by the variables $u_{ij}(t)$. For instance, for each $t \in \{1,2,3,\ldots, T\}$ we have

$$X(t) = X(0) - \sum_{\tau=0}^{t-1} (-u_{12}(\tau) - u_{14}(\tau) + u_{21}(\tau) + u_{41}(\tau)).$$

Also, the nonnegativity constraints for $A(t)$, $B(t), X(t)$ and $Y(t)$ can be expressed in terms of $u_{ij}(t)$. For instance, $X(t) \geq 0$ reduces to

$$\sum_{\tau=0}^{t-1} (-u_{12}(\tau) - u_{14}(\tau) + u_{21}(\tau) + u_{41}(\tau)) \leq X(0).$$

The capacity constraints for $A(t)$ and $B(t)$ take the form

$$\sum_{\tau=0}^{t-1} (-u_{83}(\tau) - u_{89}(\tau) + u_{38}(\tau) + u_{98}(\tau)) \leq A_{max},$$
$$\sum_{\tau=0}^{t-1} (-u_{10,7}(\tau) - u_{10,9}(\tau) + u_{7,10}(\tau) + u_{9,10}(\tau)) \leq B_{max}.$$

The second type of nodes are the road junctions. These are the nodeswhere the flows of evacuees can split or merge. In our graph these are the nodes 4,5,6 and 9. They are characterized by the fact that at any moment of time t the inflow must equal the outflow. If we consider node 4 for instance, for each $t \in \{1,2,3,\ldots, T\}$ the following equation must hold:

$$u_{14}(t-1) + u_{34}(t-1) + u_{54}(t-1) = u_{41}(t) + u_{43}(t) + u_{45}(t).$$

Here we point out that the model we develop is built on the assumption that at the road junctions, as well as at the points A, B, X and Y the flows of evacuees are directed by the authorities.

The third type of nodes are pseudo-nodes - they do not exist in the real-world net that is modelled (here - the road infrastructure), but are taken into consideration because of the discrete nature of the time in our model. These pseudo-nodes are added to the graph in order every arc to correspond to one time-quantum. In our graph these are the nodes 3 and 7. If we consider node 3 for instance, for each $t \in \{1,2,3,\ldots, T\}$ the following two equations must hold:

$$u_{43}(t-1) = u_{38}(t) \text{ and } u_{83}(t-1) = u_{34}(t).$$

If nodes 3 and 7 were not introduced, the problem has to be reformulated. One only has to take into account each arc to how many time-quanta corresponds. For instance, the equations pertaining to node A will take the form

$$A(t+1) = A(t) - u_{84}(t) - u_{89}(t) + u_{48}(t-1) + u_{98}(t)$$

for $t \in \{1,2,3,\ldots, T-1\}$.

Since we aim at evacuating all citizens of X by $\overline{t_X}$ and all citizens of Y by $\overline{t_Y}$ we impose the constraints

$$X\left(\overline{t_X}\right) = 0 \text{ and}\} \; Y\left(\overline{t_Y}\right) = 0 .$$

Expressed in terms of $u_{ij}(t)$, these two constraints take the form

$$\sum_{\tau=0}^{\bar{t}_X - 1} \left(-u_{12}(\tau) - u_{14}(\tau) + u_{21}(\tau) + u_{41}(\tau)\right) = X(0);$$

$$\sum_{\tau=0}^{\bar{t}_Y - 1} \left(-u_{21}(\tau) - u_{26}(\tau) + u_{12}(\tau) + u_{62}(\tau)\right) = Y(0).$$

We thus have a set of linear constraints (equalities and inequalities) imposed on the variables $u_{ij}(t)$ for the respective values of i and j and for $t \in \{0,1,2,3,\ldots, T\}$. Thus, a typical feasible set for a linear programming (LP) problem is determined. What we still don't have is a cost function. We can add one in several ways. First, we can take a cost function which represents some resources used for the evacuation (and we will want to minimize it), e.g. the total financial cost of the operation. Second, since all the real-world restrictions are incorporated in the constraints determining the feasible set, we can work with a fictitious cost function. In this case any LP solver will simply find a feasible point, if such point exists. If such a point does not exist, this means that, given the real-world restrictions, all citizens of X and Y can not be evacuated out of the disaster zone in time. Using a fictitious cost function has a drawback - every point in the feasible set of the LP problem corresponds to one strategy for the evacuation of towns X and Y in time. The used LP solver could find a solution, which is unacceptable in a real-world crisis - lack of evacuation activities for several time moments (because there is enough time anyway) or sending a group of evacuees to cover a loop in the roads' system. This can be avoided in many ways. One way is to formulate a cost function (to be minimized) which describes some resources (financial, temporal) used for the evacuation. An other way is to take the following cost function:

$$X\left(\overline{t_X}-1\right)+Y\left(\overline{t_Y}-1\right)\rightarrow \min$$

(let us recall that both $X\left(\overline{t_X}-1\right)$ and $Y\left(\overline{t_Y}-1\right)$ can be expressed in terms of $u_{ij}(t)$). If the minimum of this objective function happens to be zero, one can make a step further by taking the cost function

$$X\left(\overline{t_X}-2\right)+Y\left(\overline{t_Y}-2\right)\rightarrow \min$$

and adding the terminal constraint

$$X\left(\overline{t_X}-1\right)+Y\left(\overline{t_Y}-1\right)=0.$$

Continuing in this direction one can reach the first step $\hat{t}$ such that the cost function

$$X\left(\overline{t_X}-\hat{t}\right)+Y\left(\overline{t_Y}-\hat{t}\right)\rightarrow \min$$

has a positive minimized value in the presence of the additional constraint

$$\sum_{t=1}^{\hat{t}-1}\left(X\left(\overline{t_X}-t\right)+Y\left(\overline{t_Y}-t\right)\right)=0\,.$$

In this way the minimal times needed for the evacuation of both X and Y can be established.

3. An Evacuation Problem in Continuous Time

The evacuation problem discussed in the previous section can be formulated in continuous time. Under the assumption that the authorities have complete information about the dynamics of the hazardous phenomenon in the period of the planning and that they can fully control the flows of evacuees on the roads' system, one would expect to come up with an optimal control problem. A possible gain from the continuous time set up is that different ways to solve the problem numerically can hint at suitable discrete formulations which may be far from being obvious.

Considering the problem from section 2, we will have relationships with delays. These will come from the necessity to balance the in- and outflows at the road junctions. For instance, if $u_{ij}(t)$ denotes the flow sent from node i to node j at time t and if h_{ij} denotes the time needed to cover the distance from node i to node j, the balance of the in- and outflows at node 4 at any time t is given by

$$u_{14}(t-h_{14})+u_{84}(t-h_{84})+u_{54}(t-h_{54})=u_{41}(t)+u_{48}(t)+u_{45}(t)$$

(of course, here we do not need the pseudo-nodes 3 and 7). To avoid such complications, we shall formulate an evacuation problem in this section assuming that the evacuation takes place on roads which do not cross with each other. Here is the

Problem Statement:

A settlement X is threatened by a hazardous phenomenon (poisonous or radioactive cloud, flood, storm). The whole population of X has to be evacuated to three nearby safe locations (settlements) - A_1, A_2 and A_3 before X is hit by the disaster. There are three roads leading from X to each of A_1, A_2 and A_3. A precise forecast for the dynamics (the movement) of the phenomenon is assumed to exist until all the three roads are cut off. The problem is to determine the transport flows dispatched to A_1, A_2 and A_3 respectively, at each time $t \in [0,T]$ until X is hit by the hazardous phenomenon at time $T>0$. Throughout the planning horizon the residents of X (i.e. the evacuees) will be measured in "families". It is known that some of the families have their own transport (cars) while others do not. This means that the latter must be evacuated by the authorities with public transport vehicles (e.g. busses). All public vehicles are of the same type. It is also known that in terms of the roads' capacities one public vehicle is equivalent to 3 private vehicles.

At time $t=0$ in X as well as in A_i, $i=1,2,3$ there are known numbers of public vehicles and the ones in A_i, $i=1,2,3$ can be called to come to X should the need for this arise. Each of the locations A_i, $i=1,2,3$ has known accommodation capacity.

We next proceed with structuring the information so that we can build our model in the form of an optimal control problem.

Data assumed known:

- The exact location of the border of the hazardous phenomenon in $[0,T]$;
- The number of residents (i.e. the number of families) in X at $t=0$;
- The number of public transport vehicles available to the local authorities in X, A_1 , A_2 and A_3 respectively at $t=0,$ which can be used for the evacuation;
- The number of families one public vehicle can carry;
- The number of families which have their own transport;
- The capacity of each of the roads from X to A_1 , A_2 and A_3 respectively (in number of private vehicles per unit of time);
- The times needed to cover each of the roads from X to A_1 , A_2 and A_3 respectively;
- The distances from X to each of A_i , $i=1,2,3$;
- The number of families each of the locations A_1 , A_2 and A_3 can accommodate.

To be calculated from the forecast:

$t_i(x)$ - the time at which the point on the road from X to A_i at distance x from X, will be reached by the disaster, $i=1,2,3$.

To be determined from the model problem:

- The number of public transport vehicles called to leave for X from A_1 , A_2 and A_3 respectively at each $t\in[0,T]$;
- The number of private as well as the number of public transport vehicles sent from X to A_1 , A_2 and A_3 respectively at each $t\in[0,T]$.

We next introduce the notations for the state and control variables as well as for the data.

Notations

$p(t)$ - number of families in town X at time t (this is the state variable in the optimal control problem);

$u_i(t)$ - number of families (i.e. private transport vehicles) sent to town A_i at time t, $i=1,2,3$. (control variable);

$w_i(t)$ - number of public transport vehicles sent to town A_i at time *t*, $i = 1, 2, 3$ (control variable);

$y_i(t)$ - number of public transport vehicles called from town A_i at time *t*, $i = 1, 2, 3$ (control variable).

Notations for the data:

- The number of families in X at $t = 0$ is p_0;
- At $t = 0$ there are M public transport vehicles in town X as well as N_i public transport vehicles in town A_i for $i = 1, 2, 3$;
- Every transport vehicle can take r families;
- The number of families having their own transport is q;
- The capacity of the road from X to A_i is c_i, for $i = 1, 2, 3$ (number of private vehicles per unit of time);
- The times needed to cover each of the roads from X to A_i, $i = 1, 2, 3$ are $\overline{t}_i$, $i = 1, 2, 3$. We assume that $\overline{t}_1 < \overline{t}_2 < \overline{t}_3 < T$;
- The distances between town X and towns A_1, A_2 and A_3 are a_1, a_2 and a_3 respectively;
- The towns A_1, A_2 and A_3 can accommodate m_1, m_2 and m_3 families respectively.

Assuming that the traffic flows on each of the three roads (from X to A_1, A_2 and A_3 respectively) will travel with a constant velocity, we can calculate the three velocities v_1, v_2 and v_3 : $v_i = a_i / \overline{t}_i, i = 1,2,3.$

More notations

$*t_i$ is the solution to the problem

$$\min_{0 \le x \le a_i} \left\{ t_i(x) - \frac{x}{v_i} \right\}$$

for $i = 1, 2, 3$;

* if $t_i \le 0$, we set $c_i(t) = 0$ for each $t \in [0, T]$
* if $t_i > 0$, we set

$$c_i(t) = \begin{cases} c_i & \text{for } t \le t_i \\ 0 & \text{for } t > t_i \end{cases}$$

* $\tilde{t}_i$ is the solution to the problem

$$\min_{0\le x\le a_i}\left\{t_i(x)-\frac{a_i-x}{v_i}\right\}$$

for $i=1,2,3$;

* if $\tilde{t}_i\le 0$, we set $\overline{c_i}(t)=0$ for each $t\in[0,T]$
* if $\tilde{t}_i>0$, we set

$$\overline{c_i}(t)=\begin{cases} c_i & \text{for } t\le \tilde{t}_i \\ 0 & \text{for } t>\tilde{t}_i \end{cases}$$

So, taking into account the terminal constraint $x(T)=0,$ we obtain the following dynamics in $[0,T]$:

$$\dot{p}(t)=-u_1(t)-u_2(t)-u_3(t)-rw_1(t)-rw_2(t)-rw_3(t),$$
$$p(0)=p_0,$$

subject to the following control constraints:

* from the roads' capacities

$$0\le u_i(t)+3.w_i(t)\le c_i(t),$$
$$0\le y_i(t)\le \overline{c_i}(t),$$

* from the availability (or lack) of own transport

$$\int_0^T(u_1(t)+u_2(t)+u_3(t))\,dt\le q,$$
$$\int_0^T(w_1(t)+w_2(t)+w_3(t))\,dt\le (p_0-q)/r,$$

* from the accommodation capacities of A_1 , A_2 and A_3

$$\int_0^T(u_1(t)+rw_1(t))\,dt\le m_1,$$
$$\int_0^T(u_2(t)+rw_2(t))\,dt\le m_2,$$
$$\int_0^T(u_3(t)+rw_3(t))\,dt\le m_3,$$

* from the availability of transport vehicles in A_1 , A_2 and A_3 respectively

$$\int_0^{T} y_1(t)\,dt \le N_1,$$

$$\int_0^{T} y_2(t)\,dt \le N_2,$$

$$\int_0^{T} y_3(t)\,dt \le N_3,$$

* from the availability of transport vehicles in X

∘ for $t \in \left[0, \bar{t}_1\right)$

$$\int_0^{t} \left(w_1(\tau) + w_2(\tau) + w_3(\tau)\right) d\tau \le M,$$

∘ for $t \in \left[\bar{t}_1, \bar{t}_2\right)$

$$\int_0^{t} \left(w_1(\tau) + w_2(\tau) + w_3(\tau)\right) d\tau \le M + \int_0^{t-\bar{t}_1} y_2(\tau)\,d\tau,$$

∘ for $t \in \left[\bar{t}_2, \bar{t}_3\right)$

$$\int_0^{t} \left(w_1(\tau) + w_2(\tau) + w_3(\tau)\right) d\tau \le M + \int_0^{t-\bar{t}_1} y_2(\tau)\,d\tau + \int_0^{t-\bar{t}_2} y_1(\tau)\,d\tau,$$

∘ for $t \in \left[\bar{t}_3, T\right]$

$$\int_0^{t} \left(w_1(\tau) + w_2(\tau) + w_3(\tau)\right) d\tau \le M + \int_0^{t-\bar{t}_1} y_2(\tau)\,d\tau + \int_0^{t-\bar{t}_2} y_1(\tau)\,d\tau + \int_0^{t-\bar{t}_3} y_3(\tau)\,d\tau,$$

and to the terminal constraint

$$x(T) = 0.$$

As far as the cost function is concerned, we can proceed as in Section 2 - take a cost function which represents some resources used for the evacuation (and minimize it), or take a fictitious cost function, since all the real-world restrictions are already incorporated.

What we formulated in this section is a linear continuous-time optimal control problem with no pure state or mixed control-state constraints. Relaxing some of the assumptions on which the model was built can bring us to nonlinear problems. The next section is a step in this direction.

4. Possible Extensions

The sample problems discussed in Sections 2 and 3 are built on the assumptions of complete information about the hazardous phenomenon throughout the planning horizon and of full control of the transport flows. The latter of the two assumptions is clearly less realistic.

In what follows we assume that no one can be forced to leave the city unless he/she is convinced or has been persuaded to do so. We attempt to model two causes for the decision to leave. The first one is that others leave, the second one is the pressure exerted by the authorities through the media. In both models we propose, a single city is to be evacuated, its population at time t is denoted by $x(t)$ with $x(0) = x_0$ given, and the planning has to be done for the time interval $[0, T]$ where $T > 0$ is given. Since in both models we will not take care of the traffic flows outside the city, the end of the planning horizon T is the time the city is hit by the hazardous phenomenon.

In the first model the control is the total flow out of the city at time t (denoted by $u(t)$), i.e. we do not take care of the size of the outflow at each of the thoroughfares leading out of the city. The number of those who are ready to leave at time t (denoted by $y(t)$) depends on the number of those who have already left:

$$y(t) = f(x_0 - x(t)) \tag{1}$$

The function $f(\cdot)$ is data for this model and it poses a considerable challenge from point of view of the modelling. This function describes how the behaviour of some people affects the behaviour of others and it can be established usually after extensive empirical studies. It is natural to assume that $f(x) > 0$ for $x > 0$ and that $f(0) = 0$. Next, assuming that $f(\cdot)$ is differentiable, differentiating (1) and taking into account that $\dot{x}(t) = -u(t)$ we obtain

$$\dot{y}(t) = f'(x_0 - x(t))\, u(t).$$

Since we can not force anyone to leave, we have the control constraint $0 \le u(t) \le y(t)$ for $t \in [0, T]$. If the total capacity of all roads leading out of the city (depending on the dynamics of the hazardous phenomenon) is $c(t)$ at time t, the other control constraint is $0 \le u(t) \le c(t)$ for $t \in [0, T]$. Adding the terminal constraint $x(T) = 0$, we obtain the following dynamics in $[0, T]$:

$$\dot{x}(t) = -u(t),$$
$$\dot{y}(t) = f'(x_0 - x(t))\,u(t),$$
$$x(0) = x_0, \quad y(0) = 0,$$

subject to the control constraints

$$0 \le u(t) \le y(t),$$
$$(2) \qquad 0 \le u(t) \le c(t),$$

the point-wise state constraint

$$y(t) \le x(t),$$

and the terminal constraint

$$x(T) = 0.$$

What we still need is a cost function and we can proceed as in Sections 2 and 3 - minimize a cost function which represents some resources used for the evacuation, or work with a fictitious cost function, since all the real-world restrictions are already incorporated.

It is straightforward to extend the above presented model to the case in which we want to control the traffic outflows at each of the city's outlets. Imagine there are three such outlets. Then, if $u_i(t)$ for $i = 1,2,3$ is the respective outflow at each of these outlets, everywhere in the model $u(t)$ will have to be replaced by $u_1(t) + u_2(t) + u_3(t)$, except in (2), which will have to be replaced by the three control constraints $0 \le u_i(t) \le c_i(t)$ for $i = 1,2,3$ (the meaning of $c_i(t)$ is obvious).

In the second model we present here the control is the influence exerted by the authorities through the media. Let us denote its intensity at time t with $u(t)$. The number of those who are ready to leave at time t (again denoted with $y(t)$) is determined by the number of those who have already left and by the cumulative effect of the media influence up to t:

$$(3) \qquad y(t) = g_1(x_0 - x(t)) + g_2\left(\int_0^t u(t)\,dt \right).$$

As in the previous model, the functions $g_1(\cdot)$ and $g_2(\cdot)$ are data for the model and it is a challenge to establish them. Again, it is natural to assume that $g_1(x) > 0$ for $x > 0$, $g_2(z) > 0$ for $z > 0$ and that $g_1(0) = g_2(0) = 0$.

Denote $z(t)=\int_0^t u(t)dt$. Here the outflow of the evacuees from the city at time t is taken to be $f(y(t),z(t))$, i.e. it is determined (via the function $f(\cdot)$) by the ones who decide at t to leave because others have left and by the cumulative effect of the media influence up to t. As $g_1(\cdot)$ and $g_2(\cdot)$, the function $f(\cdot)$ is data for the model. We assume that $f(0,0)=0$ and that $0\le f(y,z)\le y$ for all $y\ge 0, z\ge 0$. The latter assumptions reflects the fact that the ones who leave at t can not exceed the ones who are ready to do so at t. Next, assuming that $g_1(\cdot)$ and $g_2(\cdot)$are differentiable, differentiating (3) and taking into account that $\dot{x}(t)=-f(y(t),z(t))$ we obtain

$$\dot{y}(t)=g'_1(x_0-x(t))f(y(t),z(t))+g'_2(z(t))u(t).$$

Denoting by $c(t)$ the total capacity at time t of all roads leading out of the city ($c(t)$ depends on the dynamics of the hazardous phenomenon), we have the pure state constraint $f(y(t),z(t))\le c(t)$ for $t\in[0,T]$. Also, if we denote by $r(t)$ the available (at time t) to the authorities resources (media, financial, etc.) used to urge the population to leave the city, we have the control constraint $0\le u(t)\le r(t)$. Adding the terminal constraint $x(T)=0$, we obtain the following dynamics in $[0,T]$:

$$\begin{aligned}
&\dot{x}(t)=-f(y(t),z(t)),\\
&\dot{y}(t)=g'_1(x_0-x(t))f(y(t),z(t))+g'_2(z(t))u(t),\\
&\dot{z}(t)=u(t),\\
&x(0)=x_0,\quad y(0)=0,\quad z(0)=0,
\end{aligned}$$

subject to the control constraints

$$0\le u(t)\le r(t),$$

the point-wise state constraints

$$\begin{aligned}
&f(y(t),z(t))\le c(t),\\
&y(t)\le x(t),
\end{aligned}$$

and the terminal constraint

$$x(T)=0.$$

As far as the cost function is concerned, we can proceed as in the previously discussed cases.

The two models presented in this section are an attempt to present dynamic problems related to evacuation activities in the framework of the optimal control theory. There is still to be done until models like these two can be used for decision making support in a real-world crises. One of the reasons to present them here is that we want to hint at the need to obtain estimates which will give us the behavioral functions $f(\cdot), g_1(\cdot)$ and $g_2(\cdot)$. (Work has been done in this direction. It is pointed out in [1] that the simulation tool *DYNASMART-P* integrates "... traffic-flow models, path-processing methodologies, *behavioural rules* and information-supply strategies into a single simulation-assignment framework".). Also, because of the point-wise state constraints, the two optimal control problems pose a mathematical challenge too. Of course, as was mentioned in the introduction, there are already a number of different approaches to the evacuation problems addressed here.

References

[1] L. Han, F. Yuan, S.-M. Chin, H. Hwang, Global Optimization of Emergency Evacuation Assignments, *Interfaces* **36**, No. 6, November-December 2006, 502-513

[2] H.S. Mahmassani, H. Sbayti, X. Zhou, *DYNASMART-P 1.0 User's Guide*. University of Maryland, College Park, MD, 2004

[3] J. Troutman, *Variational Calculus and Optimal Control*. Springer, 1996

[4] B. Walshon, Evacuation Planning and Engineering for Hurricane Katrina, *National Academy of Engineering Publications* **36**, No. 1, Spring 2006, available online at: http://www.nae.edu/NAE/bridgecom.nsf/weblinks/MKEZ-6MYTFN

ISBN 978-1-58603-760-4

www.iospress.nl

ISBN 978-1-58603-760-4
ISSN 1874-6268